

# DISCOVERIES FROM SATELLITE ORBITS

By D. G. KING-HELE

Royal Aircraft Establishment, Farnborough, England

*Summary*—Artificial satellites have already contributed much to our knowledge of the earth's environment, and many of these discoveries have been made merely by observing their orbits, which suffer small changes as a result of the effects of air drag and any peculiarities in gravity. This paper describes the new results in the following subjects: the shape of the earth, which is less flattened than was previously believed and is slightly pear-shaped; the average density of the upper atmosphere, which at heights of 200-700 km is much greater than was previously supposed; and the variations in upper-air density, which prove to be strongly under solar control.

## 1. *Introduction*

ARTIFICIAL satellites have provided new and powerful methods for examining the earth and its environment. Since the first satellite launching three years ago a great deal of new information has been amassed and several important discoveries have been made. Some of these discoveries—for example, the zones of radiation about the earth—have depended on measurements by instruments aboard the satellites; but much new knowledge, possibly less startling yet no less valuable, has come indirectly, by studying certain small changes in the orbits of satellites. Every satellite, whether instrumented or not, inevitably feels the effect of air drag and of any peculiarities in the earth's gravitational field. By scrutinizing the orbits and analysing the small perturbations which appear in them, the properties of the upper atmosphere at heights above about 180 km and the detailed form of the gravitational field have been elucidated in far more detail than was previously possible. This increased knowledge has also brought with it some interesting new hypotheses. This paper describes the new results about the earth's gravitational field and the upper atmosphere, and briefly discusses some of the theories which have been proposed to explain them.

## 2. *Orbital Theory*

In order to investigate the earth and atmosphere by means of orbital perturbations, we must first have a theory which shows what perturbations are caused by each particular property of the earth and atmosphere. It might be supposed that a subject with such an illustrious history

as celestial mechanics, with in the last 300 years has attracted the attention of many famous mathematicians, from Newton and Lagrange to Poincare and Einstein, would have included the parochial problem of earth-satellite orbits as a special case of some wider theory. But in fact the theory has had to be developed more or less from first principles in the last few years, because an artificial earth satellite is a very unusual astronomical object, and does not conform to the assumptions customary in celestial mechanics, for three main reasons. First, it is much closer to the parent body than any known natural satellite, so that the perturbing effects of asymmetry in the earth's gravitational field require to be analysed in far more detail than is usual: to give a numerical example, the chief effects of the earth's oblateness are greater for a near satellite than for the moon by a factor of about  $10^6$ . Second, the orbit of an artificial satellite is often inclined at a large angle to the equator, so that the small-angle approximations which are so popular in celestial mechanics cannot be applied. Third, an artificial satellite is an ephemeral object by astronomical standards: under the action of air drag, its orbit usually comes to a fiery end within a few years, in contrast to the millions of years customary in astronomy.

The effects of the earth's oblateness and air drag on the orbit of a satellite have been investigated by various authors<sup>(1-11)</sup>, and it is only possible here to give a very brief outline of the results. A fuller summary may be found elsewhere<sup>(12)</sup>.

If the earth were spherical and had no atmosphere, a satellite would move in an ellipse of fixed size and shape, lying in a plane which passed through the earth's centre and was fixed in direction in space (ignoring small effects such as those of the sun and moon). The main effect of the

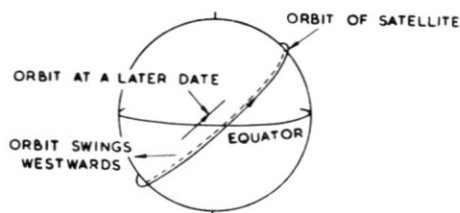


FIG. 1. The effect of the earth's oblateness on a satellite orbit.

differences in gravitational force produced by the earth's oblateness is to change the direction of the orbital in space: its inclination to the equator remains constant, but the plane rotates steadily about the earth's axis in the direction opposite to the satellite's motion, as shown in Fig. 1, at a rate of up to about 8 degrees per day. Another effect of the earth's oblateness is to make the orbital ellipse rotate in its own plane, forward

for near-equatorial orbits and backward for polar orbits. The earth's oblateness has, however, no significant effect on the size and shape of the orbit.

The effect of the atmosphere on the orbit is fortunately quite different. The atmosphere has only a minute influence on the orientation of the orbital plane and the orientation of the orbit in its own plane. Instead the main effect of air drag is to retard the satellite every time it makes its closest approach to the earth, at perigee, so that the satellite does not swing out to quite such a great distance at the subsequent apogee. Consequently the apogee height is steadily reduced, while the perigee height remains almost constant, as shown in Fig. 2. The orbit contracts and becomes more nearly circular, and from the rate of contraction the air density at heights near that of perigee can be estimated.

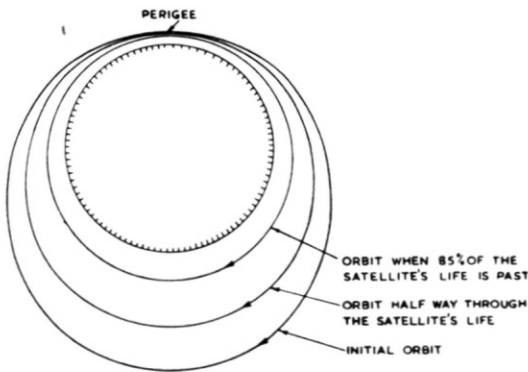


FIG. 2. Contraction of satellite orbit under the action of air drag.

There are various other, smaller perturbations to satellite orbits, due to the gravitational attractions of the sun and moon, the radiation pressure of sunlight, relativity effects and electrical drag. For most satellite orbits these effects can be regarded as small corrections, though in some rather unusual conditions (e.g. for a very elongated orbit which extends to an appreciable fraction of the moon's distance, or a balloon-satellite) one or more effects can be important.

### 3. The Figure of the Earth

3.1. *Gravitational potential*—The gravitational potential  $U$  of a spherically symmetrical body of mass  $M$  at an exterior point distant  $r$  from the centre is given by the simple formula

$$U = -\frac{gM}{r} \quad (1)$$

where  $g$  is the gravitational constant. For a nearly spherical body such

as the earth, the appropriate form for the potential is an expansion as a series of spherical harmonics; and, if the potential is assumed to be independent of longitude—an assumption which is still customary, since no reliable longitude-variation has been established—the expansion may be written in terms of the Legendre polynomials  $P_n$  as

$$U = -\frac{gM}{r} \left\{ 1 + \sum_{n=2}^{\infty} J_n \left( \frac{R}{r} \right)^n P_n(\cos \theta) \right\} \quad (2)$$

where  $R$  is the earth's equatorial radius, the  $J_n$  are constants whose values are to be determined and  $J_1$  is zero if the equatorial plane is chosen to pass through the earth's centre of mass. In equation (2),  $\theta$  is the co-latitude, and the explicit forms for the first two relevant  $P_n$  are

$$P_2(\cos \theta) = \frac{1}{2}(3 \cos^2 \theta - 1)$$

$$P_3(\cos \theta) = \frac{1}{2}(5 \cos^3 \theta - 3 \cos \theta)$$

In this expansion for  $U$  the even-numbered  $J_n$  express effects which are symmetrical about the equator, i.e. they correspond to the general flattening of the earth, and the odd-numbered  $J_n$  express any asymmetry of the earth about the equator, i.e. any tendency towards a pear-shape.

Before the advent of satellites the only one of the  $J_n$  which could be evaluated was  $J_2$ . Newton made the first estimate, from a theoretical argument, and the value was subsequently improved from measurements of arc-lengths over the earth's surface, from gravity surveys, from the motion of the moon and from the precession of the earth's axis. Before the launching of satellites the most widely accepted value<sup>(13)</sup> for  $J_2$  was  $1091 \times 10^{-6}$ ; and a conventional value of  $J_4$ , of  $-2.4 \times 10^{-6}$ , chosen to make the meridional section of the earth an exact ellipse, was sometimes used.

3.2. *Evaluation of the  $J_n$  from satellite orbits*—With the aid of the orbital theory mentioned in section 2, it is possible to express the perturbations to the orbit of a satellite moving in the gravitational field specified by equation (2) to any desired order of accuracy. A perturbation  $P$ , say, may be expressed as

$$P = \sum_{n=2}^{\infty} J_n F_n + \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} J_n J_m F_{nm} + \dots \quad (3)$$

where  $F_n$  and  $F_{nm}$  are functions of the orbital elements. In practice, the terms in the second series in equation (3) are negligible, except for the  $J_2^2$  term, since the  $J_n$  are of order  $10^{-6}$  when  $n > 2$ , and even the  $J_2^2$  term is small, so that an approximate value of  $J_2$  may be used in it. Thus, if an observed value of the perturbation  $P$  is available, a linear equation

between the  $J_n$  can be obtained. The most useful perturbation is the rate of rotation of the orbital plane about the earth's axis, which can be measured with an accuracy of about 1 in 10,000 at best. It turns out that the formula for this rate of rotation is almost independent of the odd-numbered  $J_n$ , and therefore the observed value gives a linear equation between the even-numbered  $J_n$ . Thus, if two observational values are available, from satellites having widely different orbital elements, we can obtain two linear equations between the even-numbered  $J_n$ , from which  $J_2$  and  $J_4$  can be evaluated if we are prepared to assume that  $J_6$ ,  $J_8$ , etc. are negligible. Similarly three observed values can give  $J_2$ ,  $J_4$  and  $J_6$ , if  $J_8$ ,  $J_{10}$ , etc. are assumed zero, and so on. As more satellites are observed, so it should be possible to evaluate more and more of the even  $J_n$ .

There are other perturbations which depend primarily on the odd-numbered  $J_n$ : for example, the distance of perigee from the earth's centre undergoes a small oscillation, being greater when perigee is in the southern hemisphere than when it is in the northern. From the observed values of these and other oscillations, for different satellites, it should be possible to evaluate many of the odd  $J_n$ .

By applying these methods to actual satellites the following values of the early  $J_n$  have been derived:

$$\left. \begin{aligned} J_2 &= (1082.79 \pm 0.15) \times 10^{-6} \\ J_3 &= (-2.4 \pm 0.3) \times 10^{-6} \\ J_4 &= (-1.4 \pm 0.2) \times 10^{-6} \\ J_5 &= (-0.1 \pm 0.1) \times 10^{-6} \\ J_6 &= (0.9 \pm 0.8) \times 10^{-6} \end{aligned} \right\} \quad (4)$$

The values of  $J_2$ ,  $J_4$  and  $J_6$  were obtained<sup>(14)</sup> from the rate of rotation of the orbital planes of Sputnik 2, Vanguard 1 and Explorer 7. The values of  $J_3$  and  $J_5$  were obtained<sup>(15)</sup> from the oscillatory perturbations to the orbit of Vanguard 1. The errors quoted are the standard deviations resulting from the observational errors: it is possible that somewhat larger errors may arise as a result of ignoring  $J_7$ ,  $J_8$ , etc.

The evaluation of so many of the  $J_n$  within the last two years represents a major advance in our knowledge of the earth's gravitational field. In the previous 200 years, progress was confined to obtaining a better value of  $J_2$ , and the later harmonics remained unknown. Now a much better value of  $J_2$  is available and four other of the  $J_n$  have been estimated with reasonably good accuracy.

3.3. *Deductions about the shape of the earth*—The gravitational potential is a mathematical abstraction, powerful in its generality but difficult to visualise: to bring a touch of reality into the picture it is as well to ask

what these results tell us about the shape of the earth, that is, the shape of the geoid or the mean sea-level surface.

Consider first the even harmonics, which are symmetrical about the equator. The presence of a non-zero  $J_2$  implies of course that the earth is flattened at the poles, and that its meridional cross-section is, to a first approximation, an ellipse rather than a circle. The  $J_4$  and  $J_6$  terms define the form of this cross-section in more detail and show that the shape differs very slightly from an exact ellipse.

As a rough measure of the earth's oblateness, it is convenient to introduce the flattening  $f$  of the earth, defined as the difference between equatorial and polar diameters, divided by the equatorial diameter. If we assume that the earth's sea-level surface is an exact ellipsoid, the flattening is simply related to  $J_2$ : the pre-satellite value of  $J_2$  already quoted,  $1091 \times 10^{-6}$ , gives  $1/f = 297.1$ ; the new value of  $J_2$  gives  $1/f = 298.24 \pm 0.02$ . This means that the difference between the equatorial and polar diameters is 170 metres less than was previously supposed, 42.77 km instead of 42.94 km.

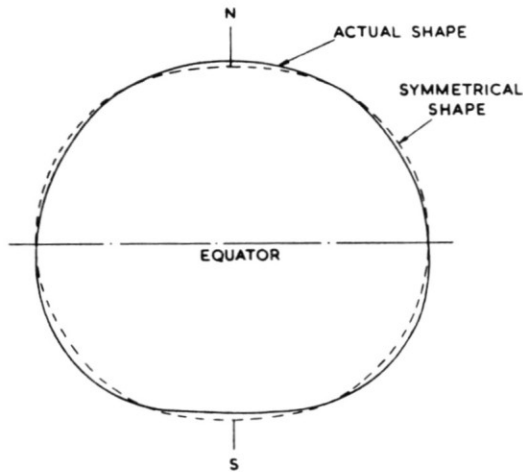


FIG. 3. Diagram showing asymmetry of the earth (not to scale).

Next, consider the odd harmonics, which express any asymmetry of the earth about the equator. The asymmetry can be described by saying that the earth shows a slight tendency towards a pear-shape, because this is the form taken by the third harmonic, and the stem of the pear is to the north, as shown in Fig. 3. This tendency is of course very slight: with the value for  $J_3$  given in equation (4), the earth's sea-level surface at the north pole is about 30 metres further from the equator than sea-level at the south pole.

The change in the value for the flattening and the discovery that the earth is not symmetrical about the equator have important implications, despite the fact that both are so small. For both go to show that the earth is not in hydrostatic equilibrium. If the earth's interior behaved like a liquid and had no internal rigidity, the flattening would be about  $1/297.3$  and the earth would be symmetrical about the equator. The asymmetry of the earth, slight though it is, does imply great stresses within the earth, and this is likely to have a profound influence on future theories of the earth's interior.

#### 4. Air density in the upper Atmosphere and its Variations

4.1. *Method*—As has been explained in section 2, the rate of contraction of a satellite orbit under the action of air drag depends primarily on the drag encountered near the perigee point. By measuring the rate of contraction of the orbit, therefore, or the rate of decrease of the orbital period, values of the air density at heights near that of perigee can be found, if the shape, weight and size of the satellite are known.

It is assumed first that the drag  $D$  on a satellite acts in the direction opposite to its velocity  $V$  relative to the ambient air, and may be expressed in the familiar form,

$$D = \frac{1}{2} \rho V^2 S C_D \quad (5)$$

where  $\rho$  is the air density,  $S$  the mean cross-sectional area and  $C_D$  a drag coefficient. Most of the satellites so far launched have rotated about an axis of maximum moment of inertia, though the direction of the axis of rotation in space has varied.  $S$  is therefore taken as the mean of the cross-sectional areas under the two extreme modes of rotation—which, for a long, thin satellite, are tumbling end over end and rotating exactly like a propeller. The value of  $S$  thus obtained may have a standard deviation of up to about 10%. The appropriate value of  $C_D$  in free molecule flow, with diffuse reflexion and a molecular speed ratio of about 6–8, is<sup>(16)</sup> near 2.2 for cylindrical and conical shapes.

It is further assumed that, above the perigee height  $y_p$ , the air density  $\rho$  varies exponentially with height  $y$ , so that

$$\rho = \rho_p \exp \{-(y-y_p)/H\} \quad (6)$$

where  $H$  is a constant, very nearly equal to what is usually called the atmospheric scale height.

It can then be shown<sup>(17)</sup> that if  $H^*$  is the best estimate of  $H$ , the air density  $\rho_A$  at a height  $\frac{1}{2}H^*$  above perigee height is given in terms of the rate of change of orbital period,  $dT/dt$ , by the equation

$$\rho_A = -\frac{0.158}{\delta} \frac{dT}{dt} \sqrt{\frac{e}{aH^*}} \left\{ 1 - 2e - \frac{H^*}{8ae} + 0 \left( e^2, \frac{H^2}{a^2 e^2} \right) \right\} \quad (7)$$

where  $a$  is the semi major axis of the orbit,  $e$  its eccentricity, and  $\delta = FSC_D/m$ ,  $m$  being the mass of the satellite and  $F$  a factor (lying between 0.85 and 1.15) which allows for the rotation of the atmosphere. If  $0.02 < e < 0.15$ , as for most satellites, and  $H^*$  does not differ from the true value of  $H$  by a factor of more than 1.5, the maximum error due to neglected terms in equation (7) is less than 5%.

4.2. *Average density*—The results<sup>(18)</sup> of applying equation (7) to 21 satellites with known orbits launched before April 1960 are shown in Fig. 4.

In discussing this diagram it is best to consider first the height-band from 180 to 300 km. In this region there are 27 points referring to satellites of different shapes and sizes, at different seasons, latitudes, etc; 22 of the points differ from the curve by a factor of less than 1.35 and none by a factor of more than 1.6. This good agreement between the results from different satellites confirms that there are no gross errors in the assumptions about, say, the cross-sectional areas, and also confirms previous evidence that the air density at these heights does not vary by a factor of more than about 1.4 as a result of variations with latitude, season, and from day to night. There is, however, one variation which can be detected in the lower part of Fig. 4: if the 24 points at heights between 180 and 250 km are split into two groups according to their dates, one group for October 1957–January 1959 and the other for August 1959–March 1960, the mean curves through each of these groups give densities which are, respectively, 10% higher and 10% lower than the curve drawn in Fig. 4. This suggests that the average density has decreased, by roughly 20%, between, say, mid 1958 and the end of 1959. The short-period variations in density, over intervals of a few days, are, as we shall see shortly, strongly under the control of the sun, the density being highest when solar radiation is at its maximum. So it is reasonable to interpret the decrease in density between mid-1958 and late 1959 as a variation during the course of the sunspot cycle: the density, like the sunspots, would have had its maximum in 1957–58, and would be expected to continue its present decline until the minimum of the sunspot cycle, in about 1965. This effect has also shown itself at higher altitudes.<sup>(29)</sup>

At heights above 300 km, the curve in Fig. 4 divides into two branches. At these greater heights it is found<sup>(18–21)</sup> that the air density is strongly dependent on the geometrical position of the sun, and varies gradually between a maximum value a little after midday and a minimum value a little after midnight. These maximum and minimum values are labelled “midday” and “midnight” in Fig. 4. The density on the “midday” curve exceeds that on the “midnight” curve by factors of 1.6, 3, 6 and 11 at heights of 400, 500, 600 and 700 km respectively. It is as if the sun “draws up”



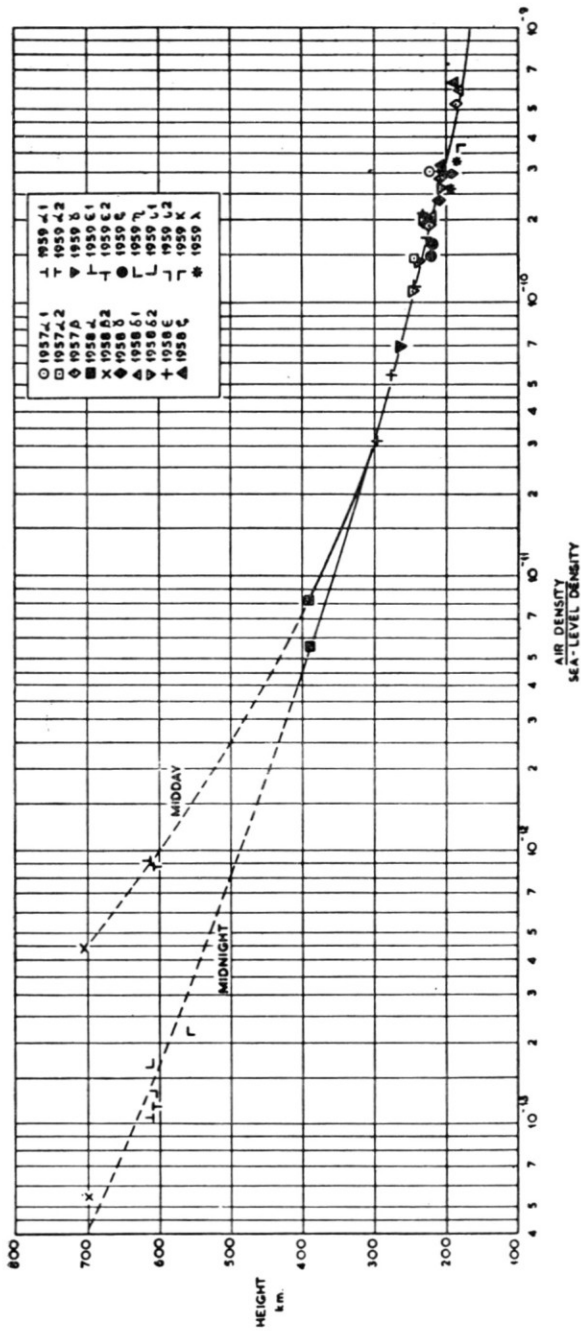


FIG. 4. Upper-atmosphere density obtained from the orbits of 21 satellites.

the atmosphere underneath it, with the surfaces of constant density exhibiting a "hump", whose height increases as the height above the earth increases. At a height of 500 km this hump is already 150 km high<sup>(21)</sup>. The significance of these findings is discussed in section 4.4.

Before the launching of satellites, the generally-accepted values of air density<sup>(22)</sup> at heights of 200–400 km were much lower than those indicated by Fig. 4. The previous values have proved to be too low by a factor which increases from 3 at 200 km to 10–15 at 400 km.

4.3. *Variations in density*—If the air density at a given height were constant from day to day, the daily decrease in orbital period of a satellite would slowly and smoothly increase as the perigee point sank lower into the atmosphere (provided there was no change in its effective cross-sectional area). In practice, for nearly all the satellites so far launched, the daily decrease in period has varied somewhat irregularly, the irregularities apparently being first noticed<sup>(23,24)</sup> with Sputnik 2.

By the end of 1958 it had become apparent that there was a pattern in the irregularities: they showed a strong tendency towards a fluctuation with a period of about 28 days. Figure 5 shows the variations in air density during the year, expressed as a fraction of the average air density. The values plotted are from Sputnik 2 and Sputnik 3 rocket<sup>(25)</sup>, both referring to heights of 200–250 km, with the gap between them bridged<sup>(20)</sup> by Vanguard 1, which refers to a height of 650–700 km. The grid lines in Fig. 5 are at intervals of 28 days and it will be noticed that the maximum values of density always occur near the grid lines, the periodicity being particularly exact towards the end of the year. The results from Vanguard 1, though applying to different heights and latitudes, fit in well with those from the Sputniks but have a larger amplitude. A comparison of the main curve in Fig. 5 with the subsidiary data at the top of the graph, which give the latitude of perigee and indicate whether or not it is in darkness, shows that the density exhibits no obvious correlation either with latitude or with time of day. We therefore conclude from Fig. 5 that the density shows a strong tendency to fluctuate once every 28 days, and that this fluctuation is not likely to depend either upon latitude or upon changes in density between day and night: it is a world-wide phenomenon, occurring over a great depth of the atmosphere.

The 28-day fluctuation is controlled by the sun, which rotates, relative to the earth, once every 27 or 28 days (the exact value depends on the solar latitude), and gives rise to well-known 27/28-day periodicities in geomagnetic records, cosmic rays and displays of the aurora. These latter effects can be envisaged as due to streams of particles which shoot out from the sun and rotate with it, rather like the spray from a revolving



water-sprinkler, and sweep across the earth at intervals of about 27 or 28 days. The periodicity is not infallible or exact, since the source of disturbance on the sun may change in position or intensity, but a tendency towards a 28-day recurrence is usually detectable. Several comparisons<sup>(19,20)</sup> between solar radiation energy on wavelengths from 3 to 20 cm and the density as indicated by satellites have confirmed the hypothesis of solar control. Figure 6 shows on one diagram the rate of decrease of orbital period for Vanguard 1 (which is a measure of the air density) and the solar radiation on a wavelength of 10.7 cm, adapted from results given by Jacchia<sup>(20)</sup>. The agreement between the two records is as good as can ever be expected from a solar-terrestrial relationship.

4.4. *Discussion*—The results of sections 4.2 and 4.3 may be summed up as follows. The air density at heights between 200 and 700 km is strongly under solar influence, and tends to exhibit a 27/28-day oscillation with an amplitude which increases from about  $\pm 15\%$  at a height of 200 km to a factor of about 1.4 at a height of 700 km (see Fig. 5) in a year when solar activity was at its maximum. Day-to-night variations in density are small at a height of 200 km, not more than  $\pm 10\%$ , but increase gradually with height, until at 700 km the maximum daytime value exceeds the minimum night-time value by a factor of about 10 (see Fig. 4). Other variations are less important.

The sun thus exerts control over the atmosphere in two different ways: the short-term variations in density at heights above 200 km are related to the solar radiation energy; the average density at heights above 400 km depends on the geometrical position of the sun. How does the sun achieve this dominance?

In considering this question, it is helpful to bring into the discussion the air temperature, which, as can easily be shown from the equation of state and the hydrostatic equation, depends on the gradient of the density, and is, to a first approximation, proportional to the "scale height"  $H$  defined in equation (6). At heights between 200 and 600 km the temperature in darkness is probably about  $1000^\circ\text{K}$ ; but in full sunlight it probably rises from about  $1200^\circ\text{K}$  at 200 km height to about  $2000^\circ\text{K}$  at 600 km height<sup>(29)</sup>. These figures are somewhat uncertain, because it is difficult to determine accurately the slopes of the curves in Fig. 4. Since the rate of decrease of  $\rho$  with height is proportional to  $\frac{1}{H}$  it follows that an increase in  $H$ , and hence in temperature, reduces the rate of decrease of  $\rho$ . Thus higher densities at a given height usually imply higher temperatures, and Fig. 4 can be interpreted as showing that the temperature is higher on the sunlit half of the earth, at heights above 300 km.

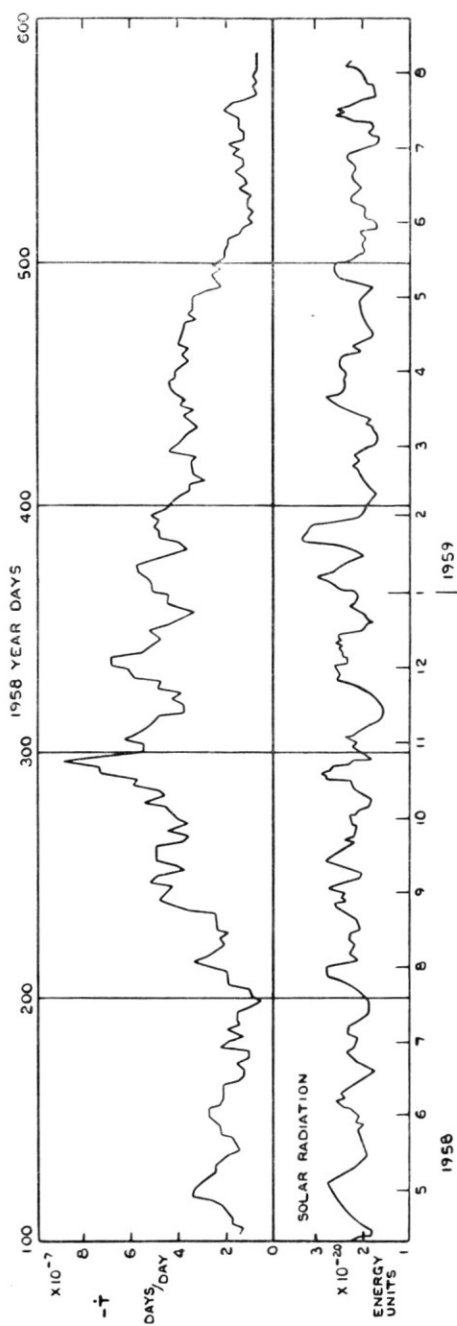


FIG. 6. The rate of decrease of period,  $-\dot{T}$ , for Vanguard I, compared with solar  $10^{-7}$  cm radiation.

The various mechanisms by which the sun might produce higher densities or temperatures have been discussed by various authors. Chapman<sup>(26)</sup> has emphasized the importance of heat conduction down from the sun's corona, which, at the earth's distance, is still at a very high temperature, about 250,000° K. Krassovsky *et al.*<sup>(27)</sup> have pointed out that the flux of electrons with energies near 10 kev, as measured by Sputnik 3, is enough to account for a considerable increase of temperature with height, and these electrons might well be of solar origin. It seems most probable, however, that, as argued by Nicolet<sup>(28)</sup>, the temperature differences are mainly due to the absorption of solar wave-radiation, probably in the far ultra-violet, at altitudes near 200 km. Whatever the detailed explanation may be, results from satellites have certainly shown that the sun has a far more important influence over our environment than was previously believed, and have lent a new plausibility to Chapman's view that the earth should be looked upon as immersed in the sun's atmosphere.

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#### DISCUSSION

J. M. REIFFENSTEIN: After the deductions of Mr. King-Hele it is evident, that the density of the upper atmosphere is in relation with the 28 days period of the rotation of the sun. An influence of the momentaneous activity of the sun, by sunspots, protuberances etc. is very probably existent.

I would like to elucidate the following points:

The sunspot-relativity number  $R$  is an anonymous number, which is calculated from the number of sunspots and groups on the whole visible surface of the sun, controlled constantly by numerous observers.

An influence on the density of the earth's atmosphere in the rhythm of the rotation of the sun can only be understood by individual sources of this action, lying in the sun,

and from which the electrically loaded particles, electrons, ions and the like are starting in their direction to the earth. As such individual sources could be considered larger single sun spots, groups of spots, protuberances and the like. These individual sources would have the maximum influence on the earth if they are located near the centre of the sun's disc visible from the earth: In this case the particles, which exhaust with great velocity radially from the sun, would immerge the earth and act in the manner of the jet of a rotating water sprinkler, as Mr. King-Hele said, and thus influence the density of the upper atmosphere.

A further confirmation of these ideas bring the curves of the diminution of the decreasing ratio of the satellite's perigee when passing through the earth's shadow, shown at last by Mr. King-Hele. Here it is quite evident, that the satellite, when passing through the earth's shadow, finds a medium of lower density: It is so as if it would come into a zone of "dead water" behind the earth.

This correlation between the density of the earth's upper atmosphere and the activity of the sun spots was deduced more than 50 years ago by the Austrian astronomer Hanns Hoerbiger, who died in 1931, and it seems to me a significant fact that just on his 100th birthday his ideas are proved true by modern science.

I propose therefore to pay extreme attention to sun spots or sunspot groups, which are located just in the centre of the visible sun's disk and which thus could exercise the maximum influence on the earth's atmosphere and secondarily cause the variation of the satellite's orbit.

D. G. KING-HELE: I should like to emphasize again that at present it seems likely that the air density is influenced more by wave radiations from the sun than by streams of particles; but, with this reservation, I agree with Dr. Reiffenstein's analysis of the matter.